HotSpot: Signed Distance Function Optimization with an Asymptotically Sufficient Condition

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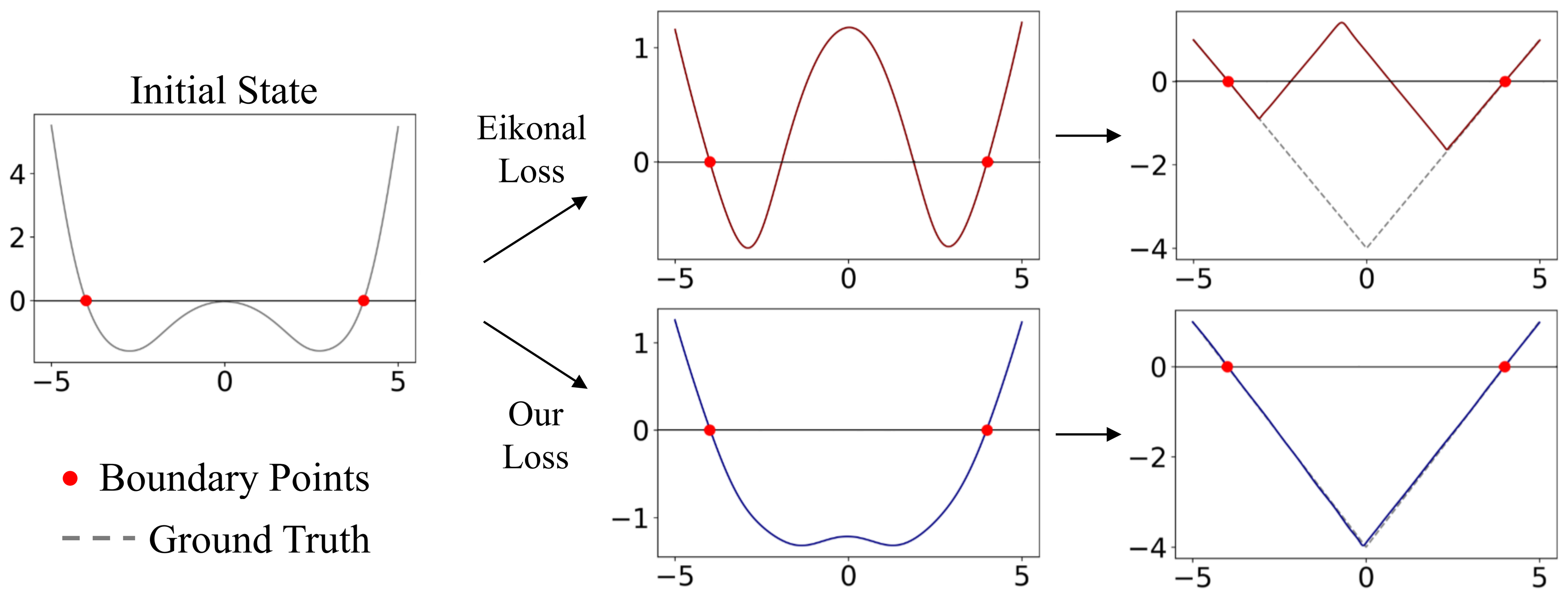
BACKGROUND

TASK Given point cloud Γ_0, train a neural network $u_\theta(\boldsymbol{x})$ to represent the signed distance to the reconstructed surface $\Gamma \supset \Gamma_0$.

CHALLENGES

- Losses from SDF necessary conditions are not sufficient constraints and cannot exclude non-SDF solutions.

$$\begin{matrix} min\{\text{heat loss}\} \\ \lambda \to \infty \end{matrix} \Leftrightarrow \begin{matrix} \text{Distance} \\ \text{Function} \end{matrix} \underset{\not\Leftarrow}{\overset{\text{a.e.}}{\Rightarrow}} \left\{ \begin{matrix} \text{Eikonal condition} \\ \text{Zero gradient divergence} \\ \text{Singular Hessian matrix} \\ \dots \end{matrix} \right.$$



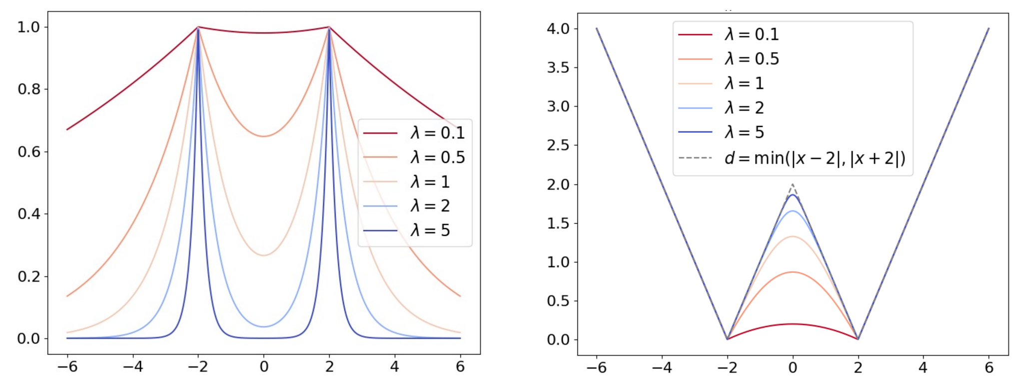
- Losses based on input point cloud distances fail near surfaces.

- Area losses used to remove redundant boundaries are often ineffective and always distort results.
- Eikonal loss makes SDF optimization unstable.

MOTIVATION

An exp-log transformation links heat simulation (via the screened Poisson equation) and distance reconstruction, addressing all four challenges.

When $\lambda \to \infty$, heat simulation provides a sufficient condition.



$$\lim_{\lambda \to \infty} \hat{d} = \lim_{\lambda \to \infty} -\frac{1}{\lambda} \log h_\lambda = d$$

METHOD

- Screened Poisson equation:

$$\begin{cases} \nabla^2 h(\boldsymbol{x}) - \lambda^2 h(\boldsymbol{x}) = 0, & \forall \boldsymbol{x} \in R^3 \setminus \Gamma \\ h(\boldsymbol{x}) = 1, & \forall \boldsymbol{x} \in \Gamma \end{cases}$$

- The minimizer of the energy $\frac{1}{2}\int \|\nabla h(\boldsymbol{x})\|^2 + \lambda^2 h(\boldsymbol{x})^2 \, \mathrm{d}\boldsymbol{x}$ is its solution.
- Transformation from signed distance $u_\theta(\boldsymbol{x})$ to heat:

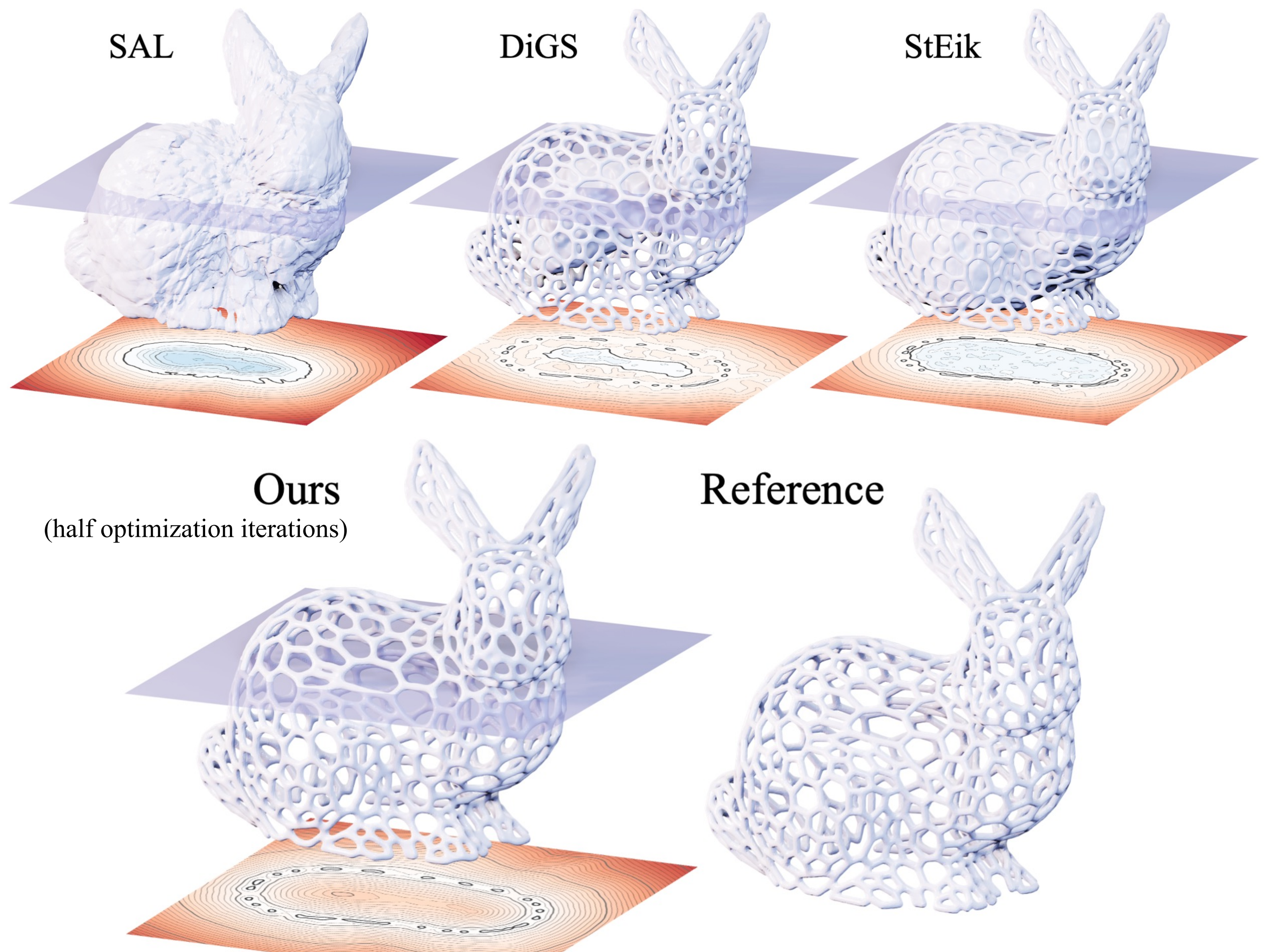
$$h(\boldsymbol{x}) = e^{-\lambda |u_\theta(\boldsymbol{x})|}$$

- Our heat loss:

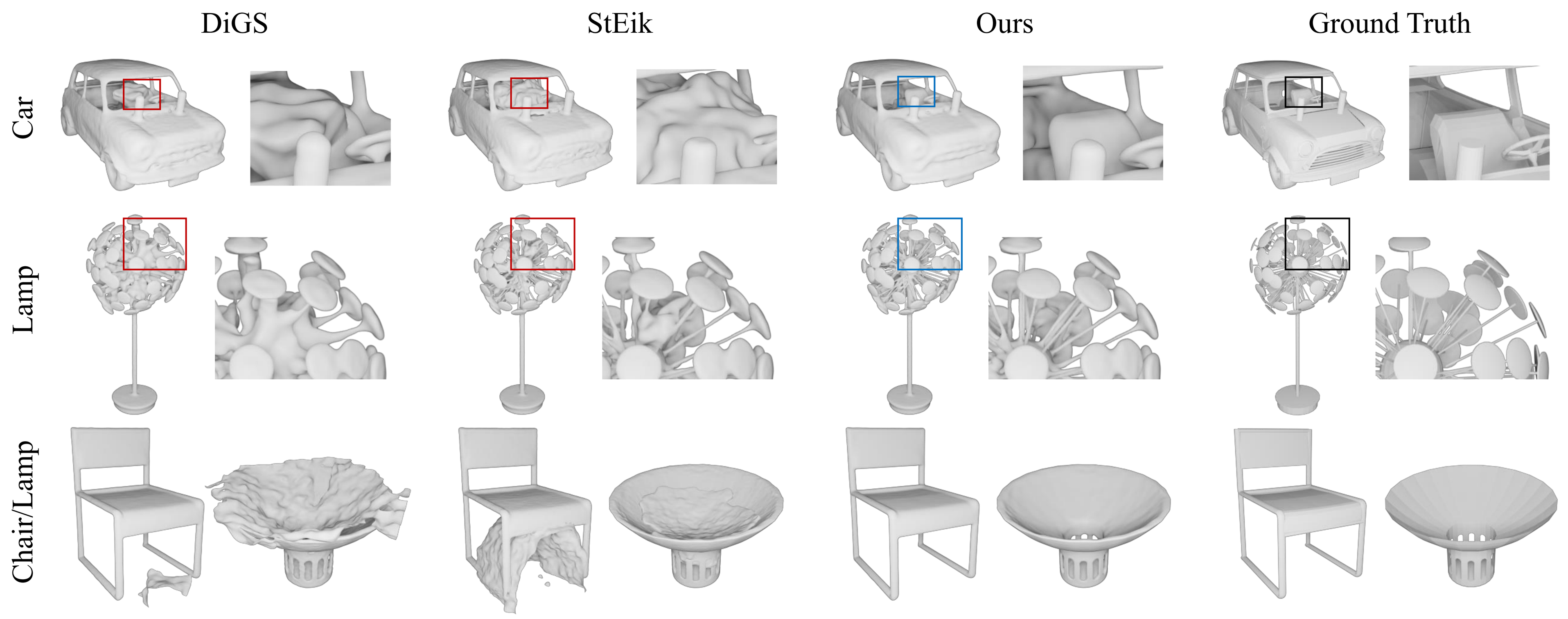
$$L_{\text{heat}} = \frac{1}{2} \int e^{-2\lambda |u_\theta(\boldsymbol{x})|} \left(\|\nabla u_\theta(\boldsymbol{x})\|^2 + 1 \right) \mathrm{d}\boldsymbol{x}$$

- Our advantages:
  1) As $1/\lambda \to 0$, the minimizer linearly converges to the distance to Γ, where $u_\theta(\boldsymbol{x}) = 0$ and $h(\boldsymbol{x}) = 1$.
  2) L_{heat} is an upper bound of surface area estimator.
  3) L_{heat} has spatial and temporal stability.

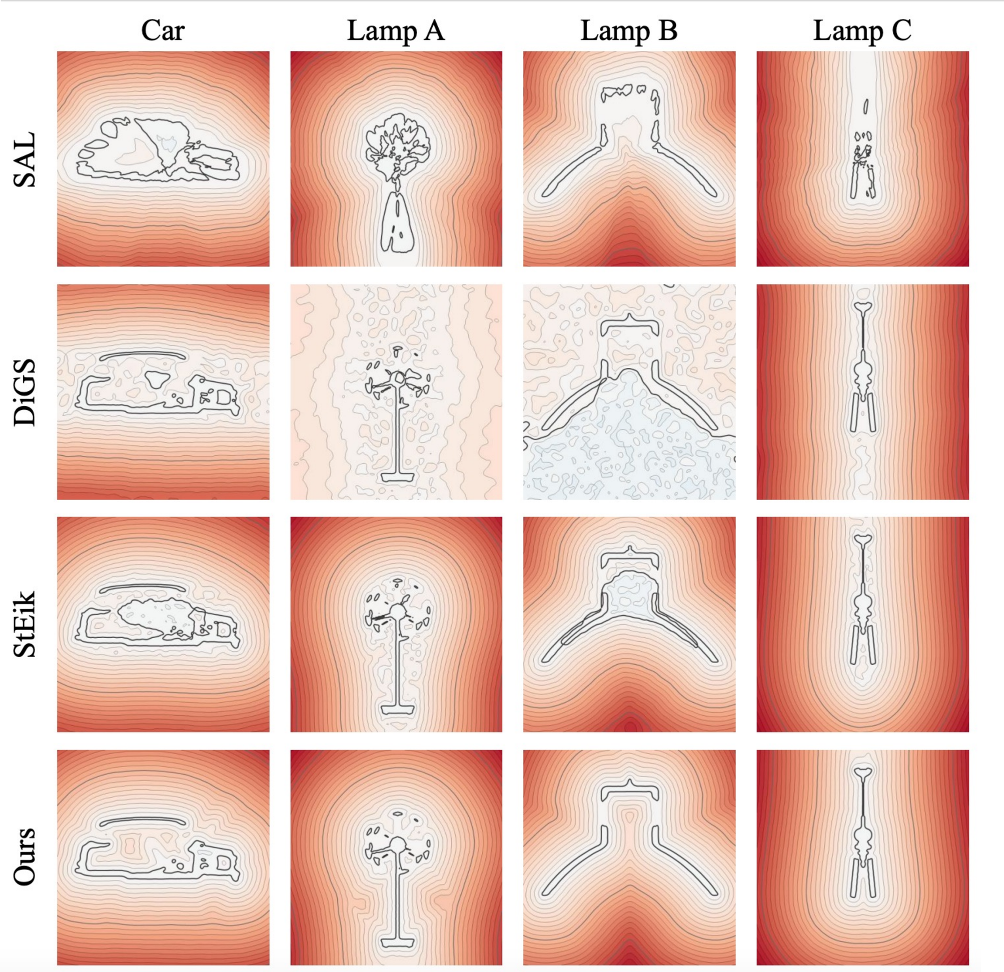
RESULTS

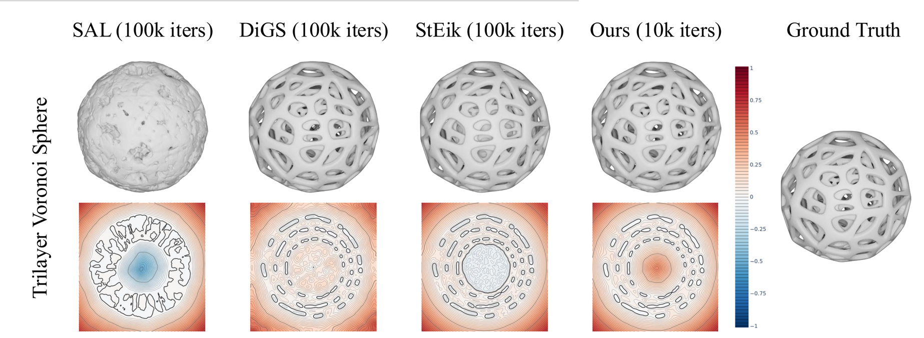


We remove extra boundaries without the area loss.

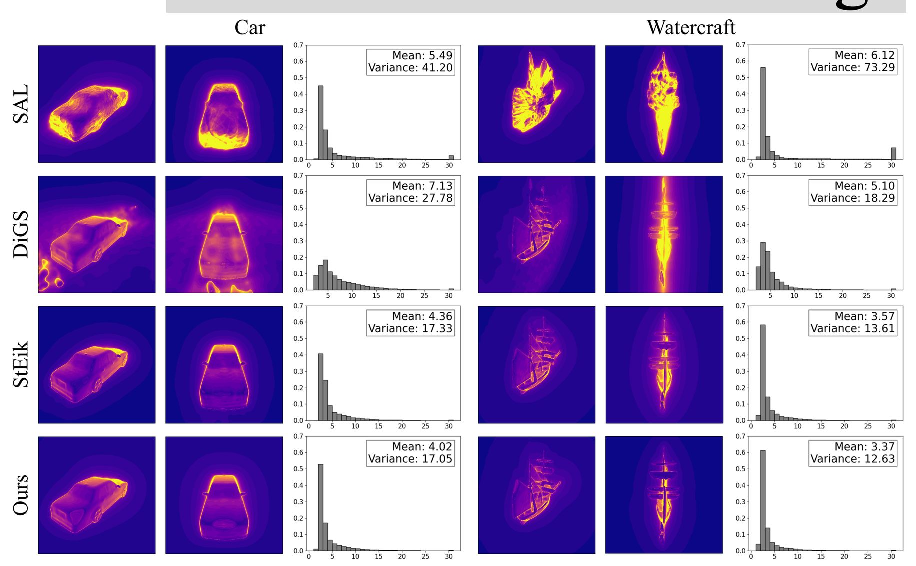


We offer better surface reconstruction and level sets.





We accelerate rendering.



We achieve high accuracy in distance queries.

	IoU ↑	d_C ↓	d_H ↓	RMSE ↓	MAE ↓	SMAPE ↓	$\text{RMSE}_{0.1}$	$\text{MAE}_{0.1}$	$\text{SMAPE}_{0.1}$
SAL	0.7400	0.0074	0.0851	**0.0251**	**0.0142**	0.1344	0.0245	0.0182	0.6848
SIREN w/o n	0.4874	0.0051	0.0558	0.5009	0.4261	1.2694	0.0513	0.0382	0.8858
DiGS	0.9636	**0.0031**	0.0435	0.1194	0.0725	0.2140	0.0152	**0.0081**	**0.1760**
StEik	**0.9641**	0.0032	**0.0368**	0.0387	0.0248	**0.0931**	**0.0147**	**0.0081**	0.1770
Ours	**0.9796**	**0.0029**	**0.0250**	**0.0281**	**0.0176**	**0.0540**	**0.0094**	**0.0047**	**0.1206**

We are faster to train.

Structure	5 × 128	5 × 256	8 × 128	8 × 256
StEik	56.5 ms/iter	106.2 ms/iter	87.8 ms/iter	170.0 ms/iter
Ours	44.9 ms/iter	84.3 ms/iter	67.9 ms/iter	128.6 ms/iter

#iters	Kangaroo	VSphere	Bunny	3-layer VSphere
SAL	20k	20k	20k	100k
DiGS	20k	20k	20k	100k
StEik	20k	20k	20k	100k
Ours	10k	10k	10k	10k

MORE